Design of an ultrafiltration unit

Tumi Maimane, Siyabonga Simelane, Zandile Keebine, Kirsten Louw, Morepe Dolamo, Dzanga Raphulu, Nkululeko Mindu, John Soko, Kyle Jacobs, Zinhle Sangweni, Koffi Messan Agbavon

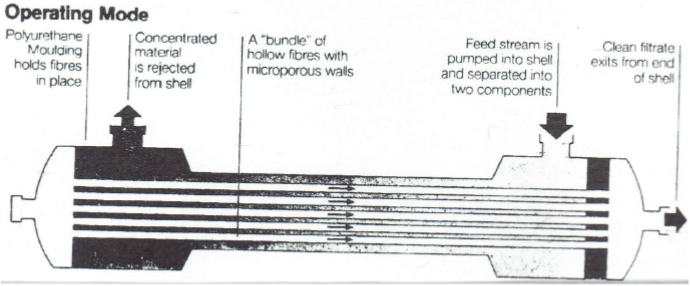
Tutors: Rahab Kgatle and Gideon Fareo

School of Computational and Applied Mathematics,
University of the Witwatersrand

January 12, 2013

Outline

Filter


Operating Mode
Polyurethane Moulding holds fibres in place
Concentrated material is rejected from shell
A "bundle" of hollow fibres with microporous walls
Feed stream is pumped into shell and separated into two components
Clean filtrate exits from end of shell

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

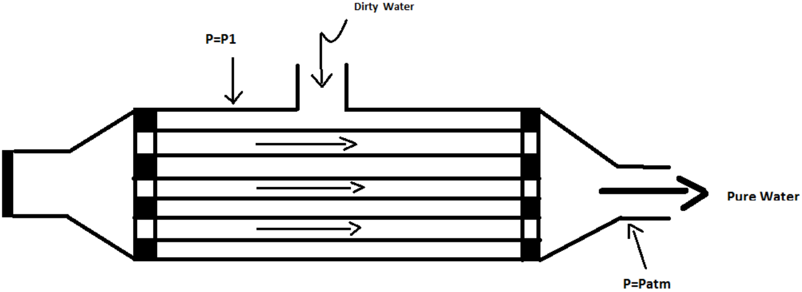
Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filter

- One of the uses of the filter is to purify water.
- Within the cartridge there are about 3000 hollow fibres made of plastic foam with radius 330μ.
- The foam is a porous material(pores with radius 0.1μ). This enables pure water to flow through the pores while filtering the small particles out.
- Impure water is pumped into the system at high pressure (100 kPa).
- The pure water moves almost radially though the porous foam. The pressure function found in the foam can be found by using D'Arcy's Law.
- Then the pure water moves axially along the lumen. Using Poiseuille Flow, one can find the velocity of the water. Hence, the flux.
- The water will then be gathered at the end of the cartridge.

Filtration Process



Cleaning cycle

- During filtration, waste accumulates and some even smaller particles get lodged into the pores.
- To clean the fibres, a high pressured (500-700kPa), short duration air pulse is sent through the lumen. Thus clearing the pores.

Cleaning cycle

- During filtration, waste accumulates and some even smaller particles get lodged into the pores.
- To clean the fibres, a high pressured (500-700kPa), short duration air pulse is sent through the lumen. Thus clearing the pores.

Objectives

- Find the expression for the flux through each lumen in terms of the length inner and outer radii.
- Determine how important parameters give maximum flux.

Objectives

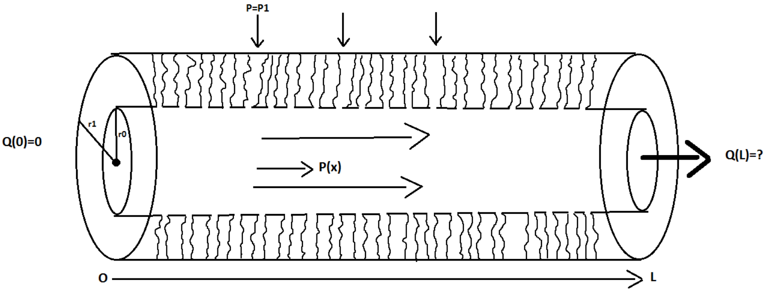
- Find the expression for the flux through each lumen in terms of the length inner and outer radii.
- Determine how important parameters give maximum flux.

Poiseuilli Flow

Poiseuilli Flow

Poiseuille flow- The steady flow of an incompressible fluid parallel to the axis of a circular pipe of infinite length, produced by a pressure gradient along the pipe.

Fibres



Navier Stokes Equation

- Navier Stokes Equation

$$\frac{\partial \vec{v}}{\partial t} + (v \cdot \nabla)\vec{v} = \frac{1}{\rho}\nabla p + \mu\nabla^2\vec{v} + \vec{F} \tag{1}$$

- Incompressibility condition for fluid

$$\nabla \cdot \vec{v} = 0 \tag{2}$$

- Velocity of fluid along the lumen

$$\vec{u} = (u_r, u_\theta, u_z) \quad (3)$$
$$= (0, 0, u_z(r)) \quad (4)$$

- From Navier-Stokes equation, three components in cylindrical are obtained:
- r- component : $\frac{\partial p}{\partial r} = 0$
- θ - component : $0 = 0$
- z-component: $\frac{dp}{dz} = \frac{\mu}{r}\frac{d}{dr}(r\frac{du}{dr})$

- Boundary condition:

$$r = r_0, u_z(r_0) = 0 \tag{5}$$

- The resulting velocity is given as :

$$u_z = \frac{(r^2 - r_0^2)}{4\mu} \frac{dp}{dz} \tag{6}$$

- Flux = volume of fluid entering a cross-section area

$$Q = \int_0^{r_0} 2\pi r u_z \, \mathrm{d}r \tag{7}$$

$$Q = -\frac{\pi r_0^4}{8\mu} \frac{dp}{dz} \tag{8}$$

D'Arcy Flow

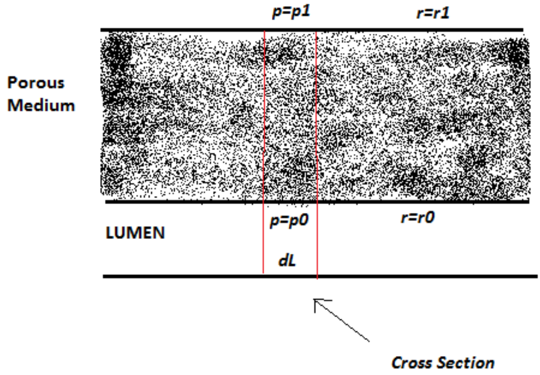
D'Arcy's Law

states that the average volumetric discharge of flow through a porous medium is directly proportional to the hydraulic gradient assuming that the flow is laminar and inertia can be neglected.

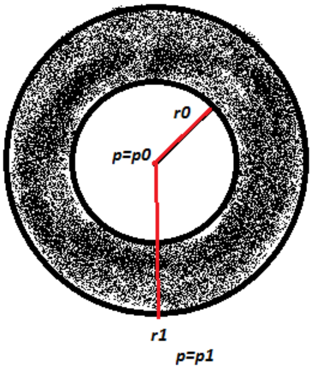
$$v_r = -\frac{k}{\mu}\vec{\nabla} p \tag{9}$$

where
K is the hydraulic permeability($2x10^{-16}m^2$)
μ is the dynamic viscosity($1x10^{-3}$Pa sec of water)
$\underline{v}$ is the volume flux/area

Longitudinal section of Fibres



Cross-section of Fibres



Cross-section area

- From the figure above we get that dA $= 2\pi r v_r$
- Due to the Continuity equation

$$\nabla \cdot (\nabla p) = 0 \tag{10}$$

- Expanding this in cylindrical coordinates gives:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dp}{dz}\right) = 0 \tag{11}$$

- Integrating we get

$$p_r = A \ln r + B \tag{12}$$

where

$$A = \frac{p_0 - p_1}{\ln(\frac{r_o}{r_1})} = 0, B = P_1 \tag{13}$$

- Boundary Conditions:

$$r = r_0 \implies p = p_0 \tag{14}$$

$$r = r_1 \implies p = p_1 \tag{15}$$

- where p_1 is a constant along the lumen and p_0 is a constant in a cross-section.

Finding the pressure in the porous medium using D'Arcy's law

- Now consider D'Arcy's law

$$V(r) = \frac{k}{\mu} P_r \tag{16}$$

- Let q(z) = total flux/length going into the lumen at a chosen cross-section.

$$q(z) = 2\pi r V(r) = constant \tag{17}$$

$$= -2\pi \frac{k}{\mu} A \tag{18}$$

- Therefore $q(z) = \left(\frac{2\pi k}{\mu} \frac{1}{\ln(\frac{r_0}{r_1})} \right) (p_0 - p_1)$
- Let $\xi = \frac{2\pi k}{\mu} \frac{1}{\ln(\frac{r_0}{r_1})}$

Section 3

- Summary of the reuslts from Poiseulli flow and Darcy's law gives :

$$Q_z(z) = q(z) \tag{19}$$

$$q(z) = -(p_1 - p(z))\xi \tag{20}$$

$$Q(z) = -\gamma \frac{dp}{dz} \tag{21}$$

- where $\gamma = \frac{\pi r_1^4}{8\mu}$ and p_0 is no longer a constant but a function $p(z)$ that varies along the lumen.

- Differentiate (15) and equating the results with (14) gives:

$$\frac{d^2p}{dz^2} - \vec{\Gamma}p(z) = -p_1\vec{\Gamma} \tag{22}$$

where $\vec{\Gamma} = \frac{\xi}{\gamma}$

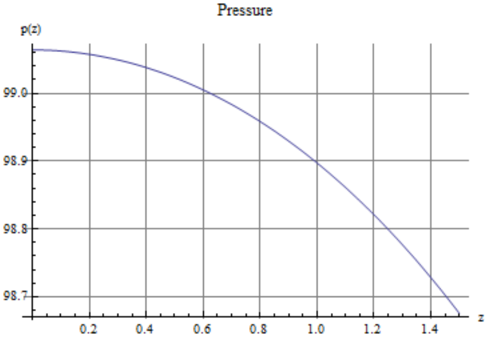
- Boundary Conditions

$$z = 0, \frac{dp}{dz} = 0 \tag{23}$$

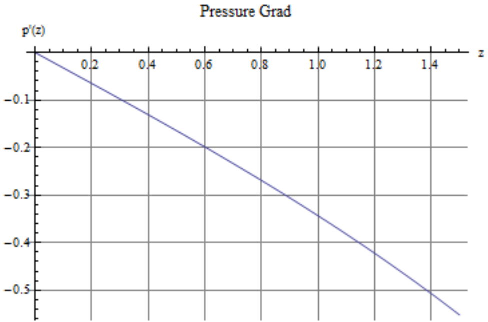
$$z = L, p = p_0 \tag{24}$$

- Pressure in the lumen : $p = p_1 - (p_0 - p_1)\frac{cosh(z\sqrt{\vec{\Gamma}})}{cosh(L\sqrt{\vec{\Gamma}})}$

Pressure



Pressure gradient



- Corresponding flux in the lumen:

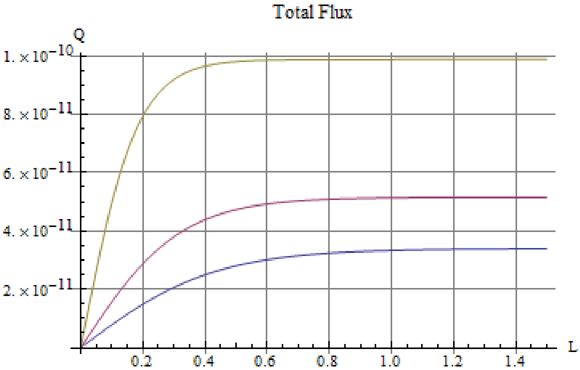
$$Q(z) = \frac{pir_1^4}{8\mu}\frac{dp}{dz} \tag{25}$$

$$= \frac{\vec{\Gamma}r_1^4}{8\mu}\left((p_0 - p_1)\frac{sinh(\sqrt{\vec{\Gamma}}z)}{cosh(L\sqrt{\vec{\Gamma}})}\right) \tag{26}$$

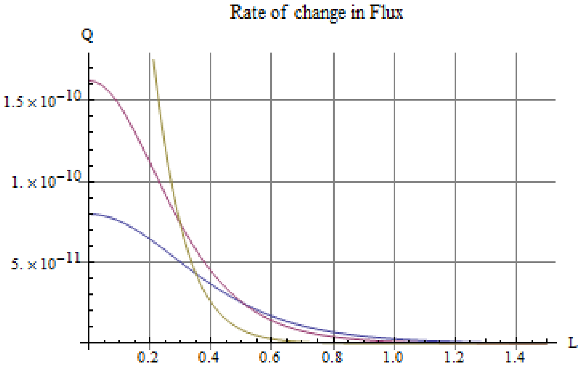
- At $z = L$ the flux is defined as follows:

$$Q(L) = \frac{\vec{\Gamma}r_1^4}{8\mu}\left[(p_0 - p_1)tanh(\sqrt{\vec{\Gamma}}L)\right] \tag{27}$$

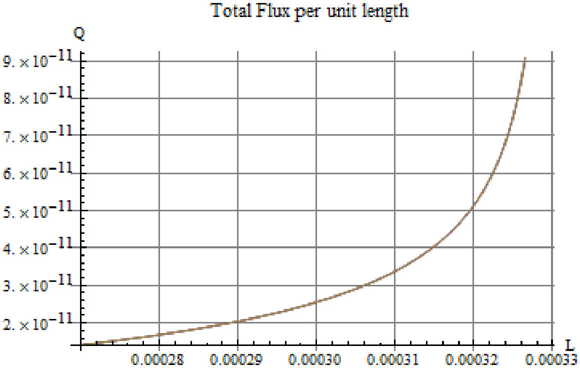
Total Flux


Total Flux
Q
1. × 10−10
8. × 10−11
6. × 10−11
4. × 10−11
2. × 10−11
0.2
0.4
0.6
0.8
1.0
1.2
1.4
L

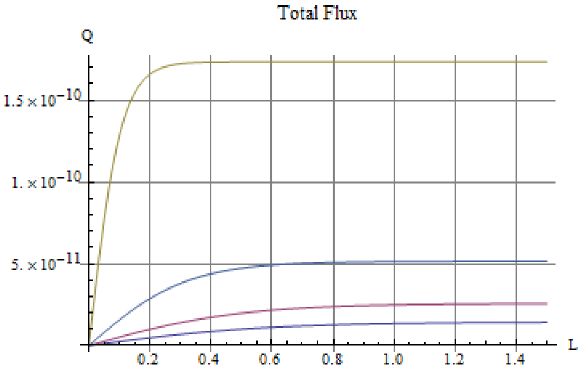
Rate of change of total flux



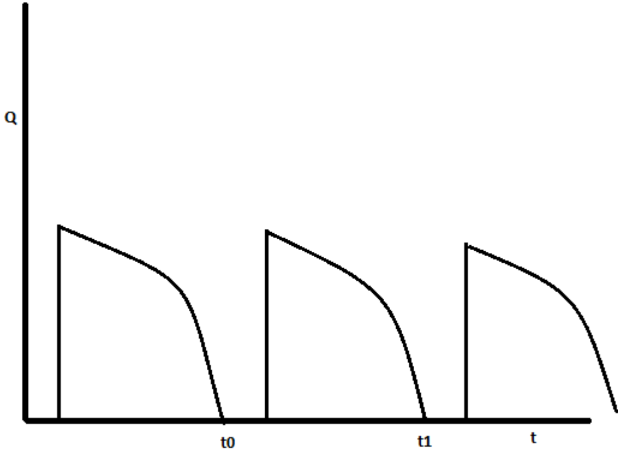
Flux and inner radius versus fixed length



Flux and inner radius versus length



Filtration cycle



Conclusion

- Problem reduction
- Identified effect of salient parameters
- Used determined effect to understand how to meet specifications of filtration unit

Q and A

Thank You!!!

Q and A

AND HAPPY BIRTHDAY DZANGA!!!

References

- Dehghan M.
  The one-dimensional heat equation subject to a boundary integral specification..
  Chaos, Solitons and fractals, 2007,Vol 32,pp.661-675.
- Dehghan M.
  A finite difference method for nonlocal boundary value problem for two-dimensional heat equation
  Applied mathematics and computation,2000,Vol.112, pp.133-142.
- Cannon JR, Lin Y and Wang S.
  An implicit finite difference scheme for the diffusion equation subject to mass specification.
  Int J Eng Sci,1990, Vol.28, pp.573-578.